Second Week.
Mathematics.

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QUESTION #1. What is a Logarithm? Of what does a logarithm consist?

ANSWER #1. The logarithm of a number is the exponent of the power to which a number, referred to as the base, must be raised to equal the given number.

A logarithm consists of a characteristic and a mantissa. The characteristic is the integral part and the mantissa is the decimal part. The mantissa is correct to as many places as are given in the table, but is never exact, because the mantissa is a never ending decimal fraction.

Any positive number except one may be taken as the base for a system of logarithms, but "10" is taken as the base for common logarithms. The log. of any multiple of "10" will be a positive whole number, and the log. of any reciprocal of a multiple of "10" will be a negative whole number.

Examples:

log. 1 = 0

log. 10 = 1	log. .1 = -1
log. 100 = 2	log. .01 = -2
log. 1000 = 3	log. .001 = -3
log. 10000 = 4	log. .0001 = -4

This makes it evident that the logarithm of a number between 10 and 100 will be the log.of 10 plus some mantissa; the log. of a number between .1 and .01 will be the log. of.01 plus some mantissa. We might say that the log. of a number between .1 and .01 is the log. of .1 minus some mantissa but,for convenience sake, the mantissa of a number is always positive, while the characteristic of a number may be either positive or negative.

From the above we can state the laws of characteristics:

The characteristic of the log. of a number greater than 1 is positive and is one less than the number of integers in the number.

The characteristic of a number between zero and one is negative and is one greater than the number of zeros between the decimal point and the first significant figure in the number.

A negative characteristic may be written in any manner that will show that it is negative in value, but for convenience in calculation it is usually represented by a positive number, less than "10", with a "-10" written after the mantissa. To find the value of the positive number less than "10", subtract the number of ciphers before the first significant figure from "9".

The mantissa of a number is always independent of the decimal point, thus if we know the mantissa of the log. of 752, we also know the log. of 75.2, 7.52, 0.000752, 752,000., etc., because the mantissa of a sequence of figures is the same no matter what the position of the decimal point is.

QUESTION #2. Find the log. of 635, 0.0005493, 3.98764

ANSWER #2. Only the mantissae of logarithms are given in the table; the characteristics are always determined by the "laws of characteristics". The characteristic is always written before the mantissa.

Using the table, we find the first three significant figures of the mantissa under the column headed "N":

To find the log. of 635: First we write down the characteristic, which, according to the laws of characteristics (The characteristic of the log. of a number greater than 1 is positive and is one less than the number of integers in the number) is "2",

Therefore:

Characteristic log. 635 = 2.

We find the first three figures of the mantissa under the column headed "N", and as there are only three significant figures in this number, we look along the same line under column "0" to find the mantissa ".80277"

We prefix the mantissa ".80277" with the characteristic "2" giving us:

log. 635 = 2.80277 Ans.

To find the log. of 0.0005493, we proceed as above, writing the characteristic first, which in this case has a negative value and is one less than the number of ciphers preceding the first significant figure, or:

Characteristic log. 0.0005493 = -4.

We find the first three significant figures "549" under column "N", looking along the same parallel line to column "3" (the next significant figure) we find the mantissa ".73981"; prefixing the characteristic as above, we have:

log. 0.0005493 = -4.73981 Ans.

To find the log. of 3.98764, we proceed as above, giving us, first, the characteristic:

Characteristic log. 3.98764 = 0.

In "N" column we find the first three significant figures "398", looking along the parallel line to column "7" we find the mantissa ".60065", but, as there are more than four significant figures, we must interpolate to find the remaining significant figures.

To interpolate, using a five figure table, we subtract the mantissa found for the first four figures from the next highest mantissa in the table and

ANSWER #2. Continued.

multiply the remainder by the difference between the number, corresponding to the lowest mantissa used, and the number of our problem, adding the product to the last figures of the lowest mantissa:

Mantissa of 398800	=	.60076	(subtracting)
" of 398700	=	.60065	
		11	mantissa difference

significant figures of number	=	398764	(sub)
" " antilog .60065	=	398700	
		64	diff.

mantissa difference	=	11	(multiplying)
number difference	=	64	
product	=	704	

lowest mantissa	=	.60065	(adding)
product of diff.	=	704	
mantissa of 398764	=	.6007204	

Therefore:

log. 3.98764 = 0.60072 Ans.

QUESTION #3. Find the antilog of: 6.58737, 0.35649, 3.65421-10, 2.65438

ANSWER #3. The number corresponding to a given logarithm is called an antilogarithm. The process of finding an antilogarithm is the reverse process of finding a logarithm.

If log. 551.2 = 2.74131, antilog.2.74131 = 551.2
To find the antilogarithm from the table, we look for the number corresponding to the given mantissa and place the decimal point according to the characteristic.
If the mantissa cannot be found in the table it is necessary to interpolate.

To find the antilog. of 6.58737, we look in the table to find the mantissa "58737" We find it in the seventh column opposite "386" so the significant figures of our antilog. will be 3867. Our characteristic "6" shows us that we will have seven integers to the left of the decimal point, and, as there are only four significant figures in our antilog, we annex three ciphers between the last significant figure and the decimal point, which gives us an antilog. of:

antilog. 6.58737 = 3,867,000. Ans.

ANSWER #3. Continued.

To find the antilog. of 0.35649, we look in the table to find the mantissa ".35649" We are unable to find this mantissa in the table so it is necessary to interpolate. To interpolate for the antilogarithm, we subtract the next lowest mantissa, in the table, ".35641" from the mantissa of our problem, ".35649" and get a difference of "8" Next we subtract the next lowest mantissa ".35641" from the next highest ".35660" and get a difference of "19" Since our characteristic is "0" the number corresponding to 0.35649 must be 8/19 of the way between antilog. 0.35641 and 0.35660, so we divide 8 by 19 to find the fifth significant figure of our antilog.:

Let x equal the antilog. we wish to find:

```
log. x       = 0.35649
log. 2.2720  = 0.35641  (next lowest mantissa)
               0.00008  difference.

log. 2.2730  = 0.35660  (next highest mantissa)
log. 2.2720  = 0.35641  (next lowest mantissa)
     0.0010    0.00019  difference.
```

Therefore log. x must be $\frac{.00008}{.00019}$ of the way between log 2.2720 and log. 2.2730, which equal .0010, so we multiply .00008 by .0010 and divide the product by .00019 :

```
      .0004̶2̶1̶          2.2720
 19 / .008000          .0004̶2̶1̶
                       2.2724 = antilog.
```

The above is shown to prove the interpolation, but to save time (which is the primary purpose of logarithms) we neglect the decimal points and find 8/19 of 1, or divide 8 by 19 and annex the quotient to the first four significant figures of the antilog.:

```
       .4̶2̶1̶
 19 / 8.000
```

Annexing the 4 to 2272, we have:

antilog. 0.35649 = 2.2724 Ans.

To find the antilog. of 3.65421-10, we proceed as above, interpolating because the mantissa ".65421" is not in the table:

ANSWER #3 Continued.

log. x = 3.65421-10
log. 4510 = 3.65418-10 (next lowest mantissa)
3 difference.

log. 4511 = 3.65427-10 (next highest mantissa)
log. 4510 = 3.65418-10 (next lowest mantissa)
9 difference.

3/9 of the difference in sequence of figures, which is 1 will be:

$9\overline{)3.00}$ = .33 = number to be annexed to lowest sequence.

Annexing 3 to 4510 gives us 45103. With a characteristic of "3.-10", which is identical to "-7", we will have six ciphers between the decimal point and the first significant figure:

Therefore:

antilog. 3.65421-10 = .00000045103 Ans.

or

$4.5103 \cdot 10^{-7}$

To find the antilog. of 2.65438, we proceed as above, interpolating:

log. x = 2.65438
log. 4512 = 2.65437
1 difference.

log. 4513 = 2.65447
log. 4512 = 2.65437
10 difference.

$10\overline{)1.0}$ = .1

Annexing 1 to 4512 gives us 45121. Our characteristic shows us that we will have "3" figures on the left of the decimal point, therefore:

antilog. 2.65438 = 451.21 Ans.

QUESTION #4. How are logs. used to perform multiplication?

ANSWER #4. The laws governing multiplication of exponents, also govern the multiplication of logarithms, because a logarithm is an exponent, therefore:
The logarithm of the product of two numbers is equal to the sum of the logarithms of the numbers.

QUESTION #5. Multiply, using logs., 150.73 x 0.641 x 0.00329656

ANSWER #5.

log. 150.73	= 2.17820	
log. 0.641	= 9.80686-10	(adding)
log.0.00329656	= 7.51806-10	
log. product	=19.50312-20 or	
	9.50312-10 or	
	-1.50312	

.50312 (mantissa of log.product)
.50311 (next lowest mantissa)
1 difference.

.50325 (next highest mantissa)
.50311 (next lowest mantissa)
14 difference.

.07 = .10 (If the second figure is less than 5 drop it, if more than 5 increase the next by 1)
14/1.00

Interpolating

Annexing 1 to the sequence of figures corresponding to the lowest mantissa, we have 31851. With a characteristic of "-1" the decimal point is placed before the first significant figure, therefore:

antilog. product = .31851 Ans.

QUESTION #6. How are logs. used to perform division?

ANSWER #6. The laws governing division of exponents, also govern division of logarithms, therefore:
The logarithm of the quotient of two numbers is equal to the log of the dividend minus the log of the divisor.

QUESTION #7. Divide, using logs.: 109.35 by 0.0037856

ANSWER #7.

log. 109.35	= 12.03882-10	(subtracting)
log. 0.0037856	= 7.57814-10	
log. quotient	= 4.46068	

The characteristic of log. 109.35 is 2. and the the characteristic of log. 0.0037856 is -2, which we changed to 7.-10 to avoid negative characteristics. 7 cannot be subtracted from 2, so we add 10 to 2, and place -10 after the mantissa, giving us a mantissa of 12.-10

mantissa.quotient	.46068	mantissa of 2889	.46075
mantissa of 2888	.46060	mantissa of 2888	.46060
	8		15

.533
15 /8.000

antilog.quotient = 28.885. Ans.

QUESTION #8. How are logs. used to perform involution?

ANSWER #8. The laws governing involution by exponents govern involution by logarithms, therefore:
To obtain the logarithm of the power of a number, multiply the logarithm of the given number by the exponent of the power to which it is to be raised.

QUESTION #9. What is the log. of 18 to the 12th power? Antilog?

ANSWER #9.

$$\begin{array}{lr} \text{log. } 18 & = 1.25527 \\ \text{exponent of power} & = \underline{\quad 12} \\ & 251054 \\ & \underline{125527} \\ \log 18^{12} & = \underline{15.06324} \end{array}$$

(multiplying) Ans.

Mantissa .06324 cannot be found in the table, so it is necessary to interpolate:

$$\begin{array}{lrl} \text{Mantissa log. } x & = .06324 & \\ \text{Mantissa } 1156 & = \underline{.06296} & \text{(next lowest mantissa)} \\ & 28 & \text{difference.} \end{array}$$

$$\begin{array}{lrl} \text{Mantissa } 1157 & = .06333 & \text{(next highest mantissa)} \\ \text{Mantissa } 1156 & = \underline{.06296} & \text{(next lowest mantissa)} \\ & 37 & \end{array}$$

$$37 \overline{)28.000000} \quad 0.756756 = .8$$

(Increasing the first figure by 1, because the next two figures are greater than 50/100 of the first figure.)

Annexing the 8 to 1156, we have the five signi--ficant figures of the antilog. "11568"

Therefore:

$$\underline{\text{Antilog. } 15.06324 \text{ or } 18^{12} = 1.1568 \cdot 10^{15} \text{ Ans.}}$$

or

$$\underline{1,156,800,000,000,000.}$$

In dividing 28 by 37, it appears that we might continue the division, since our characteristic is 15, and use the remaining figures for the remaining places on the left of the decimal point, but this is not so, because, in general the interpolation gives only one additional figure correct; that is, with a table that requires interpolation for the fifth significant figure, the sixth figure will not be correct if found by interpolation.

QUESTION #10. How are logs. used to perform evolution?

ANSWER #10. The laws governing evolution by exponents also govern evolution by logarithms, therefore:

ANSWER #10. Continued.

To obtain the logarithm of a root of a number, divide the logarithm of the number by the index of the root.

QUESTION #11. Extract the square root of 65973 and 0.065497 by logarithms. What is the 4th root of the same numbers?

ANSWER #11.

Mantissa of 65980 = .81941 (next highest)
Mantissa of 65970 = .81935 (next lowest)
6 difference.

Given number----------- = 65973
Number corresponding--- = 65970
to next lowest mantissa 3 difference.

Difference between mantissae = 6 (multiplying)
Difference between numbers = 3
18 (adding)
Next lowest mantissa = .81935
Desired mantissa = .819368 = .81937

There are five integers in the number, so the characteristic of the log. would be 4., therefore:

log.65973 = 4.81937

To find the log. of the 2nd root of 65973, we divide the log of 65973 by 2:

$$\frac{4.819370}{2} = 2.409685 = 2.40969 \text{ log.square root}$$

Mantissa 40969 cannot be found in the table so it is neccessary to interpolate for the antilog.:

Mantissa log. x = .40969
Mantissa 2568 = .40960 (next lowest)
9 difference.

Mantissa 2569 = .40976 (next highest)
Mantissa 2568 = .40960 (next lowest)
16 difference.

```
     .56 = .6
16/9.00
   8 0
   1 00
     96
      4
```

Annexing 6 to 2568 gives us 25686 as the sequence, therefore; characteristic 2 will give us three figures on the left of the decimal point:

antilog.2.40969 = 256.86 Ans. square root 65973

ANSWER #11. Continued.

To extract the square root of 0.065497, we must first find log.0.065497:

Mantissa of 65500 = .81624 (next highest)
Mantissa of 65490 = .81617 (next lowest)
7

Given number = 65497
Number next lowest = 65490
7 difference

Difference between mantissae = 7 (multiplying)
Difference between numbers = 7
49 (adding)
Next lowest mantissa = .81617
Desired mantissa = .816219 = .81622

The number 0.065497 is a decimal, and there is one cipher before the first significant figures, so the characteristic will be -2, therefore:

log.0.065497 = -2.81622

To avoid confusion, we add 10 to the characteristic and subtract 10 from it, making the log. 0.065497: 8.81622-10.

Dividing by 2 to find the square root:

$$2\overline{)8.81622-10} = 4.40811-5 = \text{log.square root}$$

Interpolating for the antilog.:

Mantissa log. x = .40811
Mantissa 2559 = .40807 (next lowest)
4 difference.

Mantissa 2560 = .40824 (next highest)
Mantissa 2559 = .40807 (next lowest)
17 difference.

```
     .23
17/4.00
   3 4
     60
     51
      9
```

Annexing 2 to 2559 gives us 25592 as the sequence; and having a characteristic of 4.-5, which is equivalent to -1, we will have no ciphers between the decimal point and the first significant figure, therefore:

antilog.-1.40811 is .25592 ANS. square root of 0.065497

ANSWER #11. Continued.

To find the 4th root of 65973, we divide the log. of 65973, which we have already, by 4:

```
    1.20484 = log. of 4th root of 65973
  4/4.81937
```

Interpolating to find the antilog.1.20484:

```
Mantissa log. x  = .20484
Mantissa 1602    = .20466 (next lowest)
                      18 difference.

Mantissa 1603    = .20493 (next highest)
Mantissa 1602    = .20466 (next lowest)
                      27 difference.
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```
          .66 = .7
    27 /18.00
        16 2
          180
          162
           18
```

Annexing 7 to 1602 gives us 16027 as the sequence, and having a characteristic of 1, we will have two integers on the left of the decimal point, therefore:

antilog.1.20484 is 16.027 Ans.4th root of 65973.

To find the 4th root of 0.065497, we divide the log. of 0.065497, which we have already, by4. The log is 8.81622-10 so we must add 30 to the characteristic and subtract 30 from it to make it divisible by 4:

```
log.0.065497 = 38.81622-40

    9.704055-10 = 9.70406 4th root,log of
  4/38.816220
```

It is not necessary to interpolate for the antilog. of 9.70406-10, because the mantissa can be found in the table.

The significant figures corresponding to the mantissa .70406 are 5059, and our characteristic is 9.-10, which is equivalent to -1. A -1 characteristic will give us no ciphers between the decimal point and the first significant figure, therefore:

antilog. -1.70406 = .5059 Ans.

4th root of 0.065497

QUESTION #12. What is a cologarithm of a number?

ANSWER #12. The cologarithm of a number is the logarithm of its reciprocal. 1/25 is the reciprocal of 25, therefore to find the value of the reciprocal, we divide 1 by 25. Using logarithms to divide we are told to subtract the logarithm of the divisor from the logarithm of the dividend, so, to find the cologarithm of a number, we subtract the logarithm of the number from the logarithm of 1. The logarithm of 1 is zero so we must increase the characteristic, of the logarithm of 1, by 10 and subtract 10 from it, to keep the value the same, in order to find the cologarithm.

QUESTION #13. What is the cologarithm of 70?

ANSWER #13. To find the cologarithm of 70, following the directions given in Answer #12, we subtract the logarithm of 70, which is 1.84510, from the logarithm of 1, whose characteristic has been changed to 10.-10:

$$
\begin{array}{lrl}
\log.\ 1 = & 10.00000-10 & \text{(subtracting)} \\
\log.\ 70 = & 1.84510 & \\
\hline
\text{colog } 70 = & 8.15490-10 & \text{which is equivalent to} \\
 & -2.15490-10 &
\end{array}
$$

In practical operation, we subtract mentally.

QUESTION #14. How is division performed using cologarithms?

ANSWER #14. Since in division of fractions, we are told to invert the divisor and multiply, dividing by a number and multiplying by its reciprocal must be identical, and since the cologarithm of a number is the logarithm of its reciprocal, adding the logarithm of the dividend to the cologarithm of the divisor, must give us the logarithm of the quotient.

There is a great advantage in the use of cologarithms since in the computation of Alternating Current problems, involving trigonometry, we have the product of two or more numbers to be divided by the product of two or more numbers. Without cologarithms, we would be required to subtract the sum of the logarithms of the divisor from the sum of the logarithms of the dividend. The cologarithm of a number is very easy to find, since we merely subtract, mentally, the logarithm of the number from the logarithm of 1, and we need only perform the one operation of adding the logarithms of the factors of the dividend to the cologarithms of the factors of the divisor to find the logarithm of the quotient, so it is quite evident that cologarithms assist materially in the consummation of the prime purpose of all logarithms, that of saving time in lengthy calculations.

Rule: To divide one number by another, using cologs., add the log. of the dividend to the colog of the divisor, for the log. of the quotient.

QUESTION #15. Using logs. and cologs. find the quotient of:

$$\frac{33.47 \times 0.04316 \times 246.6}{87.62 \times 29.68 \times 0.9585}$$

ANSWER #15. Using logs.:

log. 33.47 = 1.52466
log.0.04316 = 8.63508-10 (adding)
log. 264.6 = 2.42259
log product =12.58233-10 = 2.58233

log.87.62 = 1.94260
log.29.68 = 1.47246 (adding)
log.0.9585 = 9.98159-10
log.product =13.39665-10 = 3.39665

log.dividend=12.58233-10 (subtracting)
log.divisor = 3.39665
log.quotient= 9.18568-10 = -1.18568

The log. of the dividend is smaller than the log. of the divisor so we add 10 to the characteristic and subtract 10 from it, by placing -10 after the mantissa leaving it with the same value as before:

Interpolating for the antilog.:

Mantissa log. x = .18568
Mantissa 1533 = .18554 (next lowest)
14 difference.

Mantissa 1534 = .18583 (next highest)
Mantissa 1533 = .18554 (next lowest)
29 difference

.4~~8~~ = .5 (second figure more than .5 of the first figure, therefore first increased by 1.)

```
      .48
29/14.00
   116
    240
    232
      8
```

Annexing 5 to 1533 gives us 15335. Characteristic is -1, therefore:

antilog. -1.18568 = .15335 Ans.Quotient.using logs.

Using cologs:
colog. 87.62 equals log.1 minus log 87.62

log. 1 = 10.00000-10 (subtracting)
log 87.62 = 1.94260
colog 87.62 8.05740-10

colog. 29.68 equals log. 1 minus log. 29.68

ANSWER #15. Continued.

log. 1	=	10.00000-10	(subtracting)
log. 29.68	=	1.47246	
colog 29.68	=	8.52754-10	

colog. 0.9585 equals log.1 minus log. 0.9585

log. 1	=	10.00000-10	(subtracting)
log. 0.9585	=	9.98159-10	
colog. 0.9585	=	0.01841	

Adding the logs. of the dividend and the cologs. of the divisor:

log. 33.47	=	1.52466	
log. 0.04316	=	8.63508-10	
log. 264.6	=	2.42259	
colog. 87.62	=	8.05740-10	(adding)
colog. 29.68	=	8.52754-10	
colog. 0.9585	=	0.01841	
log.quotient	=	29.18568-30 = -1.18568	

The log. quotient obtained using cologs., is identical with log. quotient obtained using logs., therefore:

antilog. -1.18568 = .15335 Ans. Quotient using cologs.

QUESTION #16. What is meant by a positive number? A negative number

ANSWER #16. A positive number is one whose value is actually present, or above zero. A positive number is written without the positive sign before it.
A negative number is one whose value is lacking, or below zero. A negative number is never written without the minus sign before it.

QUESTION #17. Give the rules for addition of positive and negative numbers.

ANSWER #17. To add positive and negative numbers, add all the positive numbers, then add all the negative numbers, subtract the smaller from the greater and prefix the sign of the greater.

QUESTION #18. Add: -6, 8, -3, 15, and -19.

ANSWER #18. (-6)+(+8)+(-3)+(+15)+(-19)= -5 Ans.

-6 (adding negative numbers)
-3
-19
-28

8 (adding positive numbers)
15
23

-28 (subtracting smaller from greater)
23
- 5

QUESTION #19. Give the rules for subtraction of positive and negative numbers.

ANSWER #19. To subtract positive and negative numbers, change the sign of the subtrahend and proceed as in addition of positive and negative numbers, that is, subtract the smaller from the greater and prefix the sign of the greater.

QUESTION #20. Subtract: -14 from -5, -25 from 36, 42 from -32.

ANSWER #20.

$$\begin{array}{r} -5 \\ \underline{-14} \\ 9 \text{ Ans.} \end{array} \qquad \begin{array}{r} 36 \\ \underline{-25} \\ 61 \text{ Ans.} \end{array} \qquad \begin{array}{r} -32 \\ \underline{42} \\ -74 \text{ Ans.} \end{array}$$

In subtracting -14 from -5, we changed the sign of the subtrahend (-14) to 14, subtracted 5 from 14 and prefixed the sign of the greater (14) which left us 9. In subtracting -25 from 36, we changed -25 to 25, and as both were then positive, we added 36 to 25 which gave us 61. In subtracting 42 from -32, we changed 42 to -42, and as both were then negative, we added -42 to -32 which gave us -74.

QUESTION #21. Give the rules for multiplication of positive and negative numbers.

ANSWER #21. To multiply positive and negative numbers, proceed as in ordinary multiplication, prefixing the proper sign to the product according to the following rule: If the factors have like signs, the sign of the product will be positive; if the factors have unlike signs, the sign of the product will be negative.

QUESTION #22. Multiply: -7 by -5, 8 by -9, 7 by 8.

ANSWER #22. -7 x -5: 7 x 5 = 35. They have like signs, therefore: -7 x -5 = 35 Ans.

8 x -9 : 8 x 9 = 72. They have unlike signs, therefore: 8 x -9 = -72 Ans.

7 x 8 : 7 x 8 = 56. They have like signs, therefore: 7 x 8 = 56 Ans.

QUESTION #23. Give rules for division of positive and negative numbers.

ANSWER #23. To divide positive and negative numbers, proceed as in ordinary division, prefixing the proper sign to the quotient according to the following rule: If the divisor and dividend have like signs, the sign of the quotient will be positive; if they have unlike signs, the sign of the quotient will be negative.

QUESTION #24. Divide: -39 by 13, -28 by -4, 9 by -18.

ANSWER #24. $13\overline{)-39}$ = -3 Ans. $\quad -4\overline{)-28}$ = 7 Ans. $\quad -18\overline{)9.0}$ = -.5 Ans.

QUESTION #25. Draw TO transmitter.

ANSWER #25. See separate diagram.